Int. J. Heat Mass Transfer. Vol. 19, pp. 1425–1431. Pergamon Press 1976. Printed in Great Britain

A MIXING-LENGTH METHOD FOR PREDICTING HEAT TRANSFER IN ROUGH PIPES

A. P. HATTON* and P. J. WALKLATE

(Received 9 April 1975 and in revised form 10 July 1975)

Abstract—A mixing length formulation which includes a finite value at a hypothetical surface within the roughness is used to predict heat transfer in rough pipes. Several empirical parameters are required in this formulation and these were obtained by making comparisons of predictions, using a range of these parameters, with experiments. The experiments were carried out with airflow in pipes roughened internally with screw threads in the Reynolds number range 2×10^4 to 3×10^5. Simple correlations are suggested for the variation of the two most important empirical parameters namely the surface mixing length and the cavity Stanton number. Using these correlations enables the appropriate form of the energy equation for one-dimensional flow to be solved numerically in the thermal entrance region with any form of thermal boundary condition and the Prandtl numbers other than unity. Good agreement is shown for measurements in the thermal entrance region with a uniform wall heat flux.

NOMENCLATURE

A, constant in the mixing length equation;
B, constant in the mixing length equation;
C_f, friction factor $2\tau_w/\rho u_b^2$;
C_p, specific heat;
e, roughness height;
e_s, sand grain roughness size;
h, heat-transfer coefficient;
k, constant in the mixing length equation;
l, mixing length;
Nu, Nusselt number;
Pr, Prandtl number;
q, heat flux;
r, radius;
R, Reynolds number $\dfrac{u_c x}{\nu}$;
Re, Reynolds number $\dfrac{u_b^2 \cdot r_c}{\nu}$;
St, Stanton number;
St', cavity Stanton number;
t, temperature;
T, dimensionless temperature defined in text;
u, velocity;
u^+, dimensionless velocity $u \Big/ \sqrt{\left(\dfrac{\tau_w}{\rho}\right)}$;
x, distance along the pipe;
y, radial distance from wall $(= r_c - r)$;
y^+, dimensional distance from wall $\dfrac{y\sqrt{\left(\dfrac{\tau_w}{\rho}\right)}}{\nu}$.

Greek symbols

α, thermal diffusivity;
ε_m, eddy diffusivity of momentum;
ε_h, eddy diffusivity of heat;
ν, kinematic viscosity;
τ, shear stress.

*Mechanical Engineering Department, The University of Manchester Institute of Science and Technology, P.O. Box 88, Manchester M60 1QD, U.K.

Subscripts

0, at the hypothetical surface;
c, at the centre;
l, at the position from which ε_m is taken constant;
w, at the wall;
b, bulk mean value;
i, initial value.

1. INTRODUCTION

IN THIS article we will describe a semi-empirical method for predicting the flow and heat transfer in rough pipes. The method is applied to fully developed turbulent flow in the regime up to and including the fully rough situation with developing thermal boundary layers and with either uniform or axially varying boundary conditions. In principle the technique could also be applied to developing boundary layer flows.

The mixing-length model is used to describe the turbulence effects and the Van Driest [1] modification is included for the region near to the wall. The rough surface is replaced by a hypothetical surface at a position within the roughnesses where the velocity is assumed to be zero but where the mixing length, and therefore the eddy diffusivity, is finite. Earlier predictions have been confined to fully rough situations where Nikuradse's rough wall-law [2] has been used in conjunction with the integral equations of the boundary layer. The physical model suggested in this article should also be applicable in the region of transition from smooth to fully rough behaviour.

With the proposed mixing length model of the rough surface effects the velocity profiles and friction factor variation with Reynolds number can be predicted. The additional assumption of a turbulent Prandtl number enables the numerical solution of the energy equation to be carried out. However because the true surface temperature is different from the hypothetical surface temperature it is necessary at this stage to introduce the further concept of a surface Stanton number.

Several empirical parameters are required in this method but it will be shown that these can be fairly simply correlated. If these correlations are reliable then the method should enable predictions to be made of the effect of thermal boundary conditions on rough pipe heat transfer and also of the variation of Stanton number in the thermal entrance region.

Some experimental work was also carried out to establish the empirical parameters but further work on a wider range of roughness shapes will be required to establish confidence in the method. Measurements were made of velocity and temperature profiles, pressure loss and heat transfer in a set of three circular pipes with internal roughnesses of Whitworth thread form previously used by Furber and Cox [3]. The empirical constants were determined from these measurements and it will be suggested that these appear to be applicable over a wider range of situations.

2. SOME PREVIOUS WORK

Nikuradse established the form of the law of the wall for a rough surface from his experiments on pipes with sand coated internal surfaces. With close packed sand the heights of the roughness projections will not be the same as the average grain size so that it is difficult to compare results obtained with this configuration with other forms of roughness. Comparison of friction factors with other roughness forms has led to the concept of "equivalent and grain roughness". Reynolds [4] gives a table of ratios of equivalent and grain size to actual roughness height.

Early work on heat transfer with spring ring roughness elements, both widely and closely spaced, was carried out by Nunner [5] who suggested the correlation

$$Nu = 0.383 Re^{0.68} C_f^{1/m}$$

where

$$m = \left(\frac{Re}{100}\right)^{1/8}.$$

Dipprey and Sabersky [6] used a sand grain type of roughness to extend the Nikuradse form of the wall law into the transition region. They also used the concept of a surface or "cavity" Stanton number and suggested the correlation

$$\frac{C_f/2St - 1}{\sqrt{(C_f/2)}} = 5.19 e_s^{+0.2} Pr^{0.44} - 8.48.$$

Owen and Thomson [7] used the same concept and carried out experiments on flat plates. Their correlation for circular pipes is

$$1/St = u_b^+ \left(u_b^+ + \frac{1}{B} + \frac{17.8}{u_b^+}\right)$$

where

$$B = \frac{1}{\alpha}(e_s^+)^{-m} Pr^{-n}$$

and

$$\alpha = 0.52, \quad m = 0.45 \text{ and } n = 0.8.$$

Further experiments on vee shaped grooves in a square duct are described by Dawson and Trass [8] and for repeated rib type roughnesses by Webb and Eckert [9] and by Webb *et al.* [10]. All these workers correlated their results using the Dipprey and Sabersky method and this was mainly satisfactory but some variation in the power of e_s^+ was observed.

A more fundamental approach to the roughness problem was made by Jayatilleke [11] who provided correlations for a near-wall region to be used as initial values in a numerical solution.

The present article will describe a method of calculating the whole of the flow region from a hypothetical zero-slip wall position. A finite mixing length l_0 is taken at this position and a cavity Stanton number St' is required to account for the difference in temperature between the true wall and hypothetical surface. For continuous roughness correlations for these quantities will be suggested in the forms

$$l_0^+ = f(e^+)$$
$$St' = f(e^+, Pr).$$

3. THEORETICAL ANALYSIS

This solution deals with heat transfer with fully developed turbulent flow in a smooth or rough pipe with boundary conditions of uniform wall temperature, uniform wall heat flux or with axial variation of either of these quantities. The fluid physical properties are assumed constant and axial conduction and viscous dissipation in the fluid are neglected.

3.1. *Equation of motion*

In the fully developed situation

$$\frac{\tau}{\tau_0} = 1 - \frac{y}{y_c} \quad \text{or} \quad 1 - \frac{y^+}{y_c^+} \tag{1}$$

$$\tau = (\nu + \varepsilon_m)\frac{du}{dy}$$

$$\therefore \quad \left(1 + \frac{\varepsilon_m}{\nu}\right)\frac{du^+}{dy^+} = 1 - \frac{y^+}{y_c^+}. \tag{2}$$

This equation can be solved for a chosen value of y_c^+ if some specification is made for ε_m. In this calculation two regions were used.

3.1.1. *Wall region* $0 < y^+ < y_i^+$. In this region the mixing length variation proposed by Van Driest [1] was used with an additional term for the surface mixing length and with the inclusion of a second order correlation term, i.e.

$$\frac{l^+}{y_c^+} = \frac{l_0^+}{y_c^+} + \frac{ky^+}{y_c^+}\{1 - \exp(-y^+/A^+)\} - B\left(\frac{y^+}{y_c^+}\right)^2 \tag{3}$$

and

$$\varepsilon_m = l^2\left[\frac{du}{dy}\right]$$

or

$$\frac{\varepsilon_m}{\nu} = l^{+2}\left[\frac{du^+}{dy^+}\right] \tag{4}$$

substituting (4) in (2) gives

$$\frac{du^+}{dy^+} = \frac{-\frac{1}{2}+\frac{1}{2}\sqrt{[1+4l^{+2}(1-y^+/y_c^+)]}}{l^{+2}}. \quad (5)$$

Using the specification of l^+ from (3) this equation was solved by the Runge–Kutta–Mersen integration technique to obtain the variation of u^+ up to the selected y_l^+.

3.1.2. *Core region* $y_l^+ < y^+ < y_c^+$. In this region the ε_m value was assumed constant at the value reached in the wall region where $y^+ = y_l^+$.

With this simplification the u^+ variation is simply

$$u^+ = \left(y^+ - \frac{y^{+2}}{2y_c^+}\right)\left(\frac{1}{1+\varepsilon_m/v}\right) + C \quad (6)$$

where C is obtained from the values at y_l^+.

Having solved for the velocity profile the Reynolds number can be calculated since

$$Re = 2u_b^+ r_c^+ \quad (7)$$

and

$$u_b^+ = \frac{2}{r_c^{+2}}\int_0^{r_c^+} u^+ r^+ \, dr^+. \quad (8)$$

For a given pipe diameter therefore, the friction factor Reynolds number relation follows. The ε_m variation is also available for use in the energy equation solution.

3.2. *Energy equation*

With the above assumptions the energy equation is

$$u\frac{\partial t}{\partial x} = \frac{1}{r}\frac{\partial}{\partial r}\left[r\cdot(\alpha+\varepsilon_h)\frac{\partial t}{\partial r}\right]. \quad (9)$$

This equation can be non-dimensionalised as before except for t which requires a different definition depending on the boundary conditions.

(1) For uniform wall temperature $T = \frac{t-t_i}{t_w-t_i}$.

(2) For uniform wall heat flux $T = \frac{t-t_i}{q_w/\rho C_p u_c}$.

Substituting into the energy equation and allowing either t_w or q_w to be a function of x gives

$$u^+u_c^+\left\{\frac{\partial T}{\partial R}+f\cdot T\right\} = \frac{1}{r^+}\frac{2}{\partial r^+}\left[r^+\left(\frac{1}{Pr}+\frac{\varepsilon_h}{v}\right)\frac{\partial T}{\partial r^+}\right] \quad (10)$$

where

$$f\text{ (for } t_w \text{ varying)} = \frac{1}{t_w-t_i}\frac{dt_w}{dR} \quad (11)$$

$$f\text{ (for } q_w \text{ varying)} = \frac{1}{q_w}\cdot\frac{dq_w}{dR}. \quad (12)$$

This is a parabolic equation which can be solved step by step along the pipe. However, since the temperature and velocity profiles are steep close to the wall it is necessary in a numerical solution to use small increments of r^+ in this region. This can be achieved by changing the independent variable to u^+ and using equal steps of u^+.

The energy equation (10) with this transformation can be written

$$u^+u_c^+\left(\frac{\partial T}{\partial R}+f\cdot T\right)\frac{dr^+}{du^+} = \frac{1}{r^+}\frac{\partial}{\partial u^+}\left[r^+\left(\frac{1}{Pr}+\frac{\varepsilon_h}{v}\right)\frac{\partial T}{\partial u^+}\frac{du^+}{dr^+}\right] \quad (13)$$

with

$$\varepsilon_h = \frac{\varepsilon_m}{Pr_t}$$

where Pr_t is a constant turbulent Prandtl number.

The boundary conditions were:

At the wall

$$u^+ = 0, \quad \left[\frac{dT}{du^+}\right]_0 = \left[\frac{dy^+}{du^+}\right]_0 \left[\frac{-u_c^+}{\frac{1}{Pr}+\frac{\varepsilon_{m_0}}{v}\frac{1}{Pr_t}}\right]$$

for a specified heat flux or $T_w = 1$ for a specified wall temperature.

The true and hypothetical wall temperature are related by the cavity Stanton number defined as

$$St' = \frac{q_w}{(t_w-t_0)\rho C_p u_c}$$

where $q_w = q_0$.

At the centre

$$u^+ = u_c, \quad \left[\frac{dT}{du^+}\right]_c = \text{constant}.$$

The Crank–Nicholson implicit finite difference method with an arbitrary weighting factor was used to solve equation (13) with 60 equal increments of u^+. The weighting factor was taken at 0.9, although other values were tried but found to make little difference, and step lengths in the R direction were doubled after every ten steps. A typical run time on the U.M.R.C.C.*-CDC 7600 was 2 s to reach 150 diameters downstream of the step in wall heat flux or temperature.

A turbulent Prandtl number of 0.9 was used in all cases and the Stanton number was calculated from the predicted temperature profiles.

4. EXPERIMENTS

The experiments were carried out using a smooth pipe and a set of three brass pipes which were roughened with geometrically similar Whitworth screw threads machined internally. The pipe lengths were 3.65 m and nominal diameters 102 mm with Whitworth thread forms as tabled below.

Pipe	Wall thickness (from of thread)	Pitch (m)	Thread height (mm) e	e/d
No. 1.	1.83	0.423	0.208	0.0021
No. 2.	1.83	1.06	0.635–0.752	0.00615
No. 3.	5.53	2.31	1.308	0.0127

The working fluid was air and the Reynolds number range was from 2×10^4 to 3×10^5.

*University of Manchester Regional Computer Centre.

5. RESULTS

5.1. *The surface mixing length correlation*

It should be borne in mind that we are faced with determining optimum correlations for the five unknown parameters which appear in equation (3), i.e. l_0, k, A^+, B and y_l^+. However it was hoped that smooth pipe values of k, A^+ and y_l^+ would be applicable to the rough pipes. Values of $k = 0.42$ and $A^+ = 26$ were selected and are well-known smooth pipe values. When making comparisons with experiments the position of the hypothetical wall was taken at the mean height of the roughnesses.

The program for solving the equation of motion was run with various combinations of B and y_l^+ for the smooth pipe case ($l_0 = 0$). Typical runs are compared with experimental values on Fig. 1 and show that good agreement can be obtained with different combinations

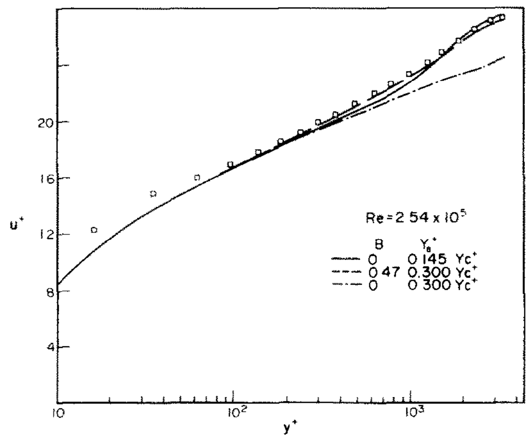

FIG. 1. Comparisons between choices of constants for smooth pipe velocity profile.

of B and y_l^+. A finite value of B gives slightly better predictions of velocity profiles but the simplification of using $B = 0$ does not result in much error. The value of y_l^+ which gave the best correlations under these conditions is $y_l^+ = 0.145 y_c^+$.

Having selected these parameters we are left with l_0 (or l_0^+). However, it is not a simple matter to correlate this parameter since it may be both roughness and Reynolds number dependent. Hopefully, therefore, a correlation of the form $l_0^+ = f(e^+)$ would seem reasonable.

The following procedure was used to establish this relationship. The program was run with a given value of l_0^+ for a range of values y_c^+. This was repeated for a range of values of l_0^+ and by comparing the predicted friction factor results with experimental values, it was possible to establish a relationship between l_0^+ and e^+. The result is shown in Fig. 2. Also shown is the same procedure applied to Nikuradse's results but using $e = e_s/2$.

Some disagreement between the present experiments and those of Furber and Cox was obtained with the pipe of smallest roughness (0.2 mm) possibly due to deposit which was observed in the base of the screw threads. This was found to be difficult to remove and could have had the effect of reducing the roughness

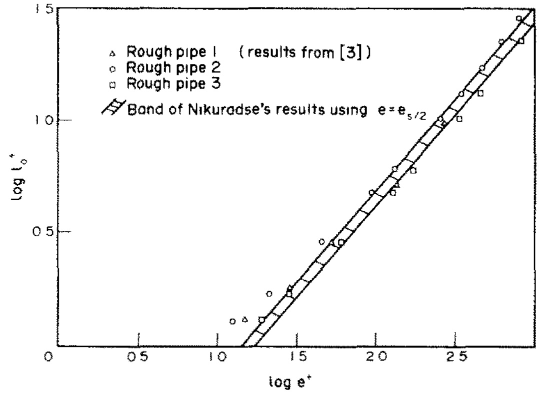

FIG. 2. Correlation for the surface mixing length variation.

heights. Because of this influence the data of Furber and Cox was used for this pipe on Fig. 2.

This curve is reasonably approximated by the expression

$$l_0^+ = C(e^+)^n$$

where $C = 0.154$, $n = 0.72$.

At the lower end some departure is apparent and it would no doubt be possible to obtain an improved fit with a more complex correlation but in view of the rather limited data this would not seem justifiable.

The authors are well aware that the procedure of obtaining a correlation from only three roughness forms is clearly questionable, but a large amount of experimental work would be necessary to obtain a general correlation. The suggested correlation accounts for the influence of the surface roughness by separating this effect from that of the main flow and should allow the calculation method to include a wider range of boundary conditions than those specifically used in establishing the correlation.

The remainder of this section will show comparisons of the predictions using the above correlation with the measurements. The additional correlation necessary for the heat transfer will be discussed below.

5.2. *Flow and friction measurements and predictions*

Figure 3 shows comparisons of measured and calculated velocity profiles for the roughest pipe. The predictions compare well and compared equally well for the other roughnesses.

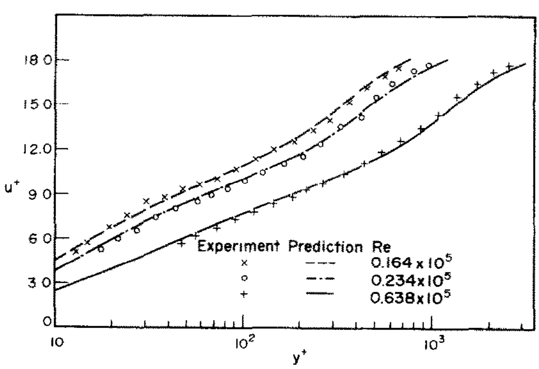

FIG. 3. Measured and predicted velocity profiles for rough pipe 3.

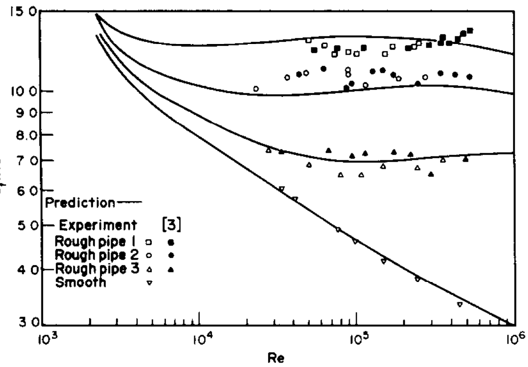

FIG. 4. Measured and predicted friction factors.

Friction factor comparisons are shown on Fig. 4 which includes the results of Furber and Cox [3]. Some difference was observed on the smallest roughness. The theoretical prediction based on the correlation of Fig. 2, which of course is based on results taken from Fig. 4, agrees well with the experimental values except in the transition region, where Nikuradse's results show a more marked reduction. The simple power law correlation for l_0^+ however does not fit well at the lower end of the points on Fig. 2 and the experimental behaviour could be more closely predicted by a better correlation in this region. However, the prediction method produces the correct physical behaviour in that the friction factors all approach the smooth pipe curve at lower Reynolds numbers.

5.3. *Heat transfer*

The concept of cavity Stanton number has been used previously by Dipprey and Sabersky as was mentioned above. It was obtained in these experiments by determining the value that was required to make the prediction agree with the experiments according to:

$$\frac{1}{St'} = \frac{1}{St\,(\text{measured})} - \frac{1}{St\,(\text{predicted})}.$$

Here the measured Stanton number is based on the wall to bulk temperature difference while the predicted value is based on the hypothetical surface to bulk difference. It follows that the cavity value St' is based on the wall to hypothetical surface difference.

The result of this procedure is shown on Fig. 5,

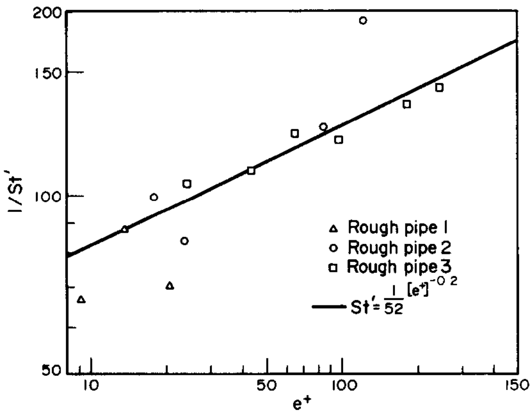

FIG. 5. Correlation for the cavity Stanton number variation.

which indicates that a correlation is possible for this quantity in the same way as for l_0^+ in the form

$$\frac{1}{St'} = 52[e^+]^{0.2}$$

or assuming the usual Prandtl number dependence

$$\frac{1}{St'} = 60[e^+]^{0.2}[Pr]^{0.4}.$$

Note also that the "cavity Stanton number" in this work is based on the bulk velocity instead of the friction velocity used by Dipprey and Sabersky.

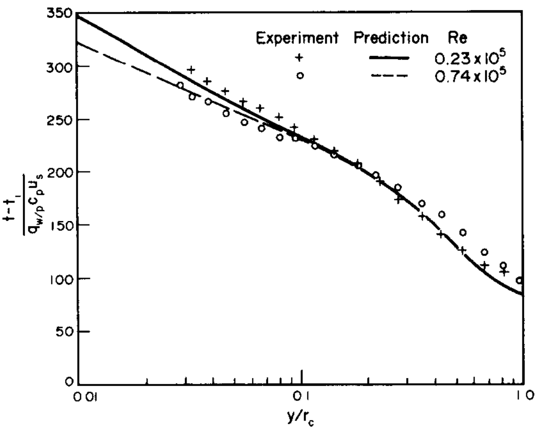

FIG. 6. Measured and predicted temperature profiles for rough pipes.

The remainder of this section is concerned with comparisons made using this correlation. Figure 6 shows temperature traverses compared with predictions and in this and all subsequent comparisons the predictions were made using the uniform wall heat flux boundary condition. In the central region only moderate agreement is shown but it should be borne in mind that in addition to the empirical constants involved in the determination of velocity and eddy diffusivity the additional assumption of a turbulent Prandtl number is required (taken as 0.9 in these predictions).

Figure 7 shows smooth pipe results for the variation of Stanton number in the entrance region. Here the results are also compared with the theoretical results of Deissler [12] and Sparrow *et al.* [13] and experimental results of Wolf and Lehman quoted in [13].

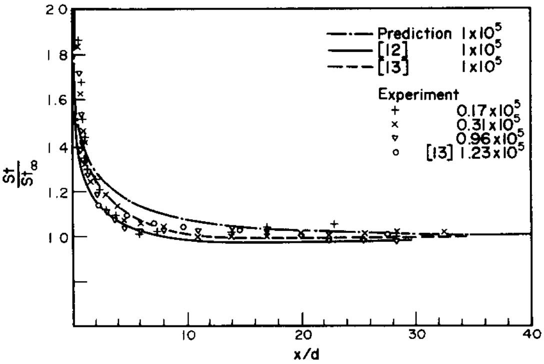

FIG. 7. Stanton number variation in the thermal entrance region of a smooth pipe.

A similar method for the velocity and eddy diffusivity predictions was used in [13] but an eigenvalue method was used for the thermal solution. These predictions are probably better than the present numerical solutions and indicate that our mixing length variation could perhaps be improved. The effect of Reynolds number is small in the entrance region. Uniform wall temperature boundary conditions were also calculated but differed only slightly from uniform wall heat flux.

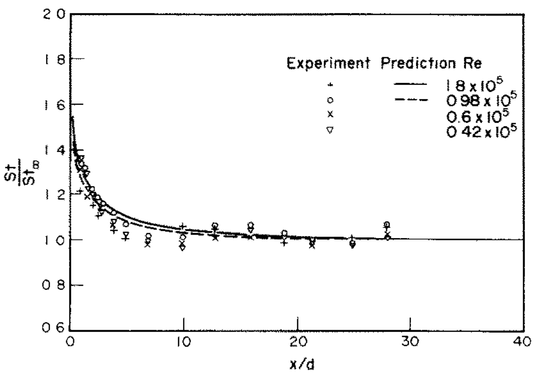

FIG. 8. Stanton number variation in the thermal entrance region for rough pipe 3.

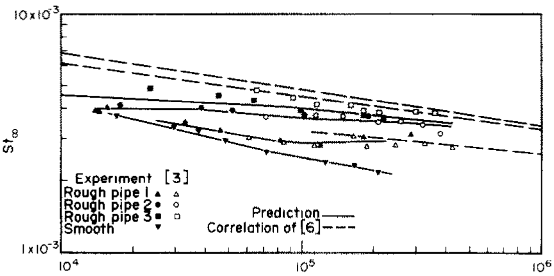

FIG. 9. Fully developed Stanton number in rough and smooth pipes.

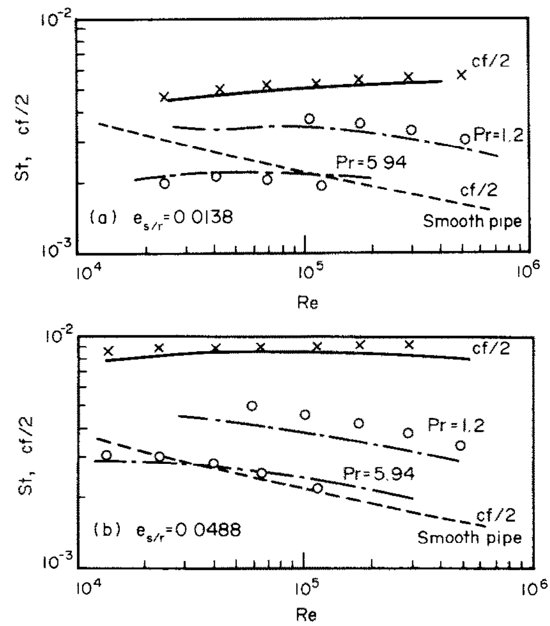

FIG. 10. Comparison of theory with experimental Stanton numbers and friction factors for sand grain roughness of Dipprey and Sabersky [6]. (a) $e_s/r = 0.0138$. (b) $e_s/r = 0.0488$.

Figure 8 shows similar predictions and measurements for the roughest pipe and it is clear that the variation of Stanton number is the same in the entrance region as the smooth pipe.

Figure 9 shows the fully developed Stanton number variations with Reynolds number for all the rough pipes. Dipprey and Sabersky's correlation is also shown for comparison but gives results rather higher than those measured. Finally, as a test of the prediction method, the program was run for four of the situations measured in [6]. The agreement, shown on Fig. 10, is good, particularly as regards the effect of Prandtl number.

To summarise it would appear that the suggested correlations offer the possibility of calculating the pressure loss and heat transfer in pipes with continuous roughness and with thermal boundary conditions of any type including axial variation of heat flux or temperature.

CONCLUSIONS

1. A prediction method has been proposed for flow and heat transfer in rough pipes in which the rough surface is replaced by a hypothetical surface of zero velocity at which there is a finite value of the mixing length.

2. The method involves the choice of a number of empirical constants for which satisfactory choices of corresponding values appear to be the same as those suitable for smooth pipes.

3. The additional correlations required for rough pipes are the surface mixing length and the cavity Stanton number variations. For distributed roughness these are

$$l_0^+ = 0.154[e^+]^{0.72}$$

and

$$\frac{1}{St'} = 60[e^+]^{0.2}[Pr]^{0.4}.$$

4. With these choices predictions of velocity profiles, friction factors, temperature profiles and Stanton number variations in the thermal entrance region show fairly good agreement with values measured in screw thread roughnesses and in sand grain roughnesses where the roughness heights are taken respectively as equal to the thread heights and half the sand grain size.

Acknowledgements—The authors are grateful to the Nuclear Power Group Limited who kindly donated the set of rough pipes and to A. C. Mugglestone and M. R. Heikal who assisted in the work.

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PREVISION DU TRANSFERT DE CHALEUR DANS LES CONDUITES RUGUEUSES A L'AIDE D'UNE METHODE DE LONGUEUR DE MELANGE

Résumé—Afin de déterminer le transfert de chaleur dans les conduites rugueuses, on utilise une hypothèse de longueur de mélange qui présente une valeur finie sur une surface hypothétique située au niveau des rugosités. Cette formulation nécessite l'introduction de plusieurs paramètres empiriques, ces derniers étant obtenus en effectuant la comparaison des prévisions avec les expériences sur un ensemble de valeurs de ces paramètres. Les expériences ont été effectuées pour un écoulement d'air dans des tubes corrugués à l'aide d'enroulements en hélice dans un domaine de nombres de Reynolds allant de 2.10^4 à 3.10^5. Des relations simples sont proposées pour les variations des deux paramètres empiriques les plus importants: la longueur de mélange sur la surface et le nombre de Stanton des cavités. L'emploi de ces relations permet de résoudre numériquement l'équation d'énergie pour un écoulement unidirectionnel dans la zone d'établissement du régime thermique pour des conditions aux limites quelconques et pour des nombres de Prandtl différents de l'unité. Un bon accord a été obtenu avec les mesures dans la zone d'établissement du régime thermique à flux pariétal constant.

EINE MISCHUNGSWEGLÄNGEN-METHODE ZUR BESTIMMUNG DES WÄRMEÜBERGANGS IN RAUHEN ROHREN

Zusammenfassung—Zur Bestimmung des Wärmeübergangs in rauhen Rohren wird eine Mischungsweglänge verwendet, welche so formuliert ist, daß sie an einer hypothetischen Oberfläche innerhalb der Rauhigkeiten endliche Werte annimmt. Zur Formulierung dieser Mischungsweglänge werden sieben empirische Parameter benötigt; diese werden unter Verwendung einer Reihe solcher Parameter durch Vergleich von berechneten Werten mit Versuchswerten ermittelt. Die Versuche wurden in luftdurchströmten Rohren durchgeführt, welche Rauhigkeiten in Form eines Schraubengewindes aufwiesen. Die Reynoldszahl wurde von 2×10^4 bis 3×10^5 variiert. Es werden einfache Korrelationsformeln zur Darstellung der Abhängigkeit von den beiden wichtigsten empirischen Parametern, nämlich der Mischungsweglänge an der Oberfläche und der Stantonzahl in den Rauhigkeitsvertiefungen, vorgeschlagen. Für den Fall der eindimensionalen Strömung ermöglichen diese Beziehungen eine numerische Lösung der entsprechenden Energiegleichung für den thermischen Einlaufbereich für jede Form der thermischen Randbedingungen und für von 1 abweichende Prandtlzahlen. Bei konstanter Wärmestromdichte an der Rohrwand ergibt sich im thermischen Einlaufbereich eine gute Übereinstimmung mit Meßwerten.

ПРИМЕНЕНИЕ МЕТОДА ПУТИ СМЕШЕНИЯ ДЛЯ РАСЧЕТА ТЕПЛООБМЕНА В ШЕРОХОВАТЫХ ТРУБАХ

Аннотация — Метод пути смешения, в котором используется конечная величина длины пути смешения на некоторой гипотетической поверхности при наличии шероховатости, используется для расчета теплообмена в шероховатых трубах. При таком подходе требуется определить несколько эмпирических параметров, и они были найдены путем сравнения расчетов в диапазоне этих параметров с экспериментальными данными. Эксперименты проводились для течения воздуха в трубах, шероховатость внутри которых нанесена в виде винтовой резьбы, в диапазоне числа Рейнольдса от 2×10^4 до 3×10^5. Предложены простые зависимости для описания изменений двух наиболее важных эмпирических параметров, а именно: значения длины пути смешения на поверхности и числа Стантона для полости. Эти зависимости позволяют численно решить уравнение энергии в соответствующем виде для одномерного течения на входном тепловом участке при любом тепловом граничном условии и числах Прандтля, отличных от единицы. Показано хорошее согласование измерений для теплового начального участка с равномерным распределением потока тепла на стенке.